Q.P. Code : 39159

(3hours) [Total marks: 80]

N.B. 1) Question No. 1 is compulsory.
2) Answer **any Three** from remaining
3) Figures to the right indicate full marks

1. a) Find Laplace transform of $f(t) = e^{-t} \sin t . cos2t$. 5

b) Show that the set of functions $\cos nx, n = 1,2,3 \dots \dots$ is orthogonal on $(0,2\pi)$. 5

c) The equations of lines of regression are $x + 2y = 5$ and $2x + 3y = -8$.
Find i) means of x and y, ii) coefficient of correlation between x and y. 5

d) Evaluate $\int_c (z^2 - 2\bar{z} + 1)dz$ where C is the circle $|z| = 1$. 5

2. a) Using convolution theorem, find the inverse Laplace transform of 6

$$F(s) = \frac{1}{(s^2+9)(s^2+4)}$$

b) Obtain Fourier series of $f(x) = |x|$ in $(-\pi, \pi)$ 6

c) Find the bilinear transformation which maps the points $z = 1, i, -1.$ onto the points $w = i, 0, -i$. Hence, find the image of $|z| < 1$ onto the w-plane. 8

3. a) If $v = e^x siny$, prove that v is a harmonic function. Also find the corresponding harmonic conjugate function and analytic function. 6

b) Using Bender –Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, subject to the conditions,

$u(0,t) = 0, u(5,t) = 0, u(x,0) = x^2(25 - x^2)$ taking $h = 1$, for 3 minutes. 6

c) Using Residue theorem, evaluate

i) $\int_0^{2\pi} \frac{d\theta}{2+cos\theta}$ ii) $\int_0^{\infty} \frac{dx}{x^2+1}$ 8

[TURN OVER]

4. a) Solve by Crank –Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$,

$u(0,t) = 0,\ u(1,t) = 2t,\ u(x,0) = 0$ taking $h = 0.25$ for two-time steps. 6

b) Obtain the Taylor's and Laurent series which represent the function

$f(z) = \frac{z}{(z-1)(z-2)}$ in the regions, i) $|z| < 1$ ii) $1 < |z| < 2$ 6

c) Solve $(D^2 - 3D + 2)y = 4e^{2t}$ with $y(0) = -3$, $y'(0) = 5$ where $D \equiv \frac{d}{dt}$ 8

5. a) Find an analytic function $f(z) = u + iv$, if 6

$$u = e^{-x}\{(x^2 - y^2)\cos y + 2xy \sin y\}$$

b) Find the Laplace transform of $\frac{\sin at}{t}$. Does the L.T of $\frac{\cos at}{t}$ exist? 6

c) Obtain half range Fourier cosine series of $f(x) = x, 0 < x < 2$. Using Parseval's identity, deduce that – 8

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

6. a) Obtain Complex form of Fourier series of $f(x) = e^x, -1 < x < 1$ 6

b) Fit a straight line to the following data, 6

x	100	120	140	160	180	200
y	0.45	0.55	0.60	0.70	0.80	0.85

c) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in form $y = a\sin(\pi x / l)$ from which it is released at a time $t = 0$. If the vibrations of a string is given by $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, show that the displacement of a point at a distance x from one end at time t is given by $y(x,t) = a \sin(\pi x / l) \cos(\pi c t / l)$. 8
